

MIDTERM EXAMINATION

April 2012 • Duration: 90 minutes

SUBJECT: CALCULUS II for BT	
Acting Head of Dept of Mathematics:	Lecturer:
Assoc. Prof. Dr. Nguyen Dinh	Dr. Janet Harris

INSTRUCTIONS: Documents, textbooks, dictionaries and calculators allowed.

Laptops and mobile phones not allowed. Exchange of documents and equipment not allowed.

SECTION A: Answer ALL questions (total 60 marks)

A1. [15 marks] Given matrices

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 0 & -2 \\ -1 & 2 & 5 \end{bmatrix},$$

find (if they exist), **(a)** $A + A^T$, **(b)** AB , **(c)** A^{-1} .

A2. [12 marks] Use a matrix method to solve the following system of equations. (Show all your working.)

$$\begin{aligned} 6x + 4y &= 2 \\ 3x - 5y &= -34 \end{aligned}$$

A3. [13 marks] Evaluate the integral

$$\iint_R 4x^2 - 3y \, dA, \quad R = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq x\}.$$

A4. [20 marks] Consider the function $f(x, y) = 8x - 12y^2 - xy^3$.

(a) Find all the first and second order partial derivatives of f .

(b) Show that f has one critical point and determine whether it is a local maximum, local minimum or saddle point.

*** PLEASE TURN OVER ***

SECTION B: Answer any TWO of the three questions (Total 40 marks)
(If you answer more than two questions, the examiner will select two answers at random. You should cross out any work which you do not wish to be marked.)

B1. [20 marks] The density of seawater depends on the temperature and the salinity (salt content). The table below gives values of density $\rho(T, S)$ for different values of temperature T (in $^{\circ}\text{C}$) and salinity S (in parts per thousand).

- (a) Estimate the value of $\rho_T(10, 32)$ and interpret its meaning.
 (b) Find an approximate expression for the linearization of $\rho(T, S)$ at $(10, 32)$.

T \ S	30	31	32	33	34
6	23.62	24.44	25.22	26.00	26.77
8	23.36	24.15	24.93	25.73	26.50
10	23.07	23.85	24.62	25.42	26.17
12	22.75	23.51	24.27	25.07	25.82

B2. [20 marks] Region R of the xy plane is the triangle with vertices at $(0, 0)$, $(1, 1)$ and $(0, 2)$. Find the volume lying under the surface $z = e^{x+y}$ and above the region R .

B3. [20 marks] A lake contains 100 fish, comprising three species: A, B and C. They are fed with two types of food: I and II. Each day a fish of species A consumes 4 units of food I and 2 units of food II. Fish B consume 1 unit of each type of food per day. Fish C eat only food II, consuming 5 units a day. It is observed that each day the fish consume a total of 135 units of food I and 220 units of food II. Represent this situation using an augmented matrix, and solve it to find the number of fish of each species in the lake.

* * * END OF QUESTION PAPER * * *